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Properties of scalar field propagators in de Sitter space

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1 Introduction

This work is devoted to the properties of quantum field theory on the strong background gravitational fields. The first reason why these phenomena deserve consideration is as follows: recent cosmological observations suggest the existence of a positive cosmological constant , which, however, is quite small to significantly affect local (on scales smaller than the galaxy clusters sizes) processes. However, in the early universe, the situation probably was different and the rate of expansion was much faster. In the latter case gravitational fields may dramatically change the properties of quantum fields as compared to those in flat space. In fact, there is still no consensus about the consequences of the peculiarities of infrared behavior of quantum fields in the de Sitter space on the one hand [1–6], on the other hand [7–11]. Moreover, even in a flat space ($R = 0$), there are nontrivial quantum effects: for example, the Unruh one (also known as the Fulling–Davies–Unruh effect)[12–14].

As background gravitational fields we consider two space-times. The first, and the most interesting for us, because it is a model describing the expanding Universe, is the static de Sitter patch discovered by W. de Sitter in 1917 [15]. Although cosmologists usually use the spatially flat Poincaré coordinates [16–18] (see also [17–19])., we will focus on static coordinates because they admit time-like killing vector, which however is not globally defined. On the horizon it becomes light-like. As will be shown below, near the horizon, the metric of the static patch is approximately equals to that of the Rindler space.[1] For this reason, the second metric that we consider is that of the Rindler space. And we want to see in details the similarities and differences of these two cases.

As it is mentioned before, both spaces under consideration have horizons – light-like surfaces. There are well-known classic works about quantum field theory in spaces with horizons. For example, the previously mentioned [12–14]. Also the relation between the existence of a bifurcate Killing horizon and some special equilibrium thermal state [20], or close relation between event horizons and thermodynamics [21]. In all, there is so-called canonical temperature $_C$ associated with the properties of geometry at the horizon.

When studying fields on the background spaces with horizons, historically, most attention was paid to either thermal states with *canonical temperature* or vacuum states [14, 22, 23]. However, there are other time translation invariant states. The main purpose of this work is to study the properties of the thermal states of a scalar field with the arbitrary temperature , and we are especially interested in different from the canonical one. The question we would like to address is whether or not one can place a gas of exact modes with arbitrary temperature in de Sitter space of high curvature.

Technical part of the work is as follows: we consider static solution of Einstein's equations in empty space (space with a cosmological constant will also be called empty), and consider the quantum field on the background of this gravitational field. Note that we are neglecting backreaction. As quantum field we consider massive real scalar field minimally coupled with gravity. Also we consider only free fields without self-interaction.

We construct propagators for thermal states with various , and study their properties. Particular attention paid to the propagators and stress-energy tensors near the horizons. It is shown that there is a dramatic difference between the two cases $= {}_C$ and $\neq {}_C$. Moreover, for $= {}_C$ the propagators depend on the geodesic distance between two points, while states with other temperatures break the symmetry of the space.

This work is based on articles [24, 25], also similar problems are discussed [26].

We want to emphasize that we will consider the de Sitter space with *small radius* ,

[1] However, one should not forget that in the Rindler space Riemann tensor is zero, while in the de Sitter space, on the contrary, the curvature is constant and non-zero at every point including the horizon.

or in other words with large Hubble constant. Otherwise, there is no reason to consider exact modes. In de Sitter space of large radius one should consider point-like particles. Similarly, in Rindler space, we will consider the large proper acceleration. Also it may be worth to stress here that only the two-dimensional case is considered to avoid technical difficulties that arise in higher dimensions. However there are reasons to assume that conclusions drawn from calculations in two dimensions are the same as in higher dimensions, because some calculations were done in an arbitrary dimension and the obtained results are essentially the same.

2 Quantum fields in the de Sitter space

2.1 Geometry of the de Sitter space

One can visualise the two-dimensional de Sitter space as the one-sheeted hyperboloid embedded in a three dimensional ambient Minkowski space (see Fig. 1):

$$dS_2 = \{X \in R^3;\ X \cdot X = X_0^2 - X_1^2 - X_2^2 = -R^2\}. \qquad (2.1)$$

(capital X denote the coordinates of a given Lorentzian frame of the ambient spacetime).

Figure 1: The de Sitter space with unit radius embedded in the three dimensional ambient Minkowski space.

The de Sitter space is the maximally symmetric solution of Einstein gravitational field equations with constant curvature, the two dimensional de Sitter space with radius R has the following curvature:

$$g^{\mu\nu} R_{\mu\nu} = \frac{2}{R^2},$$

and the isometry group of the de Sitter space is the Lorentz group of $(2+1)$ dimension ambient space $O(2;1)$. However, when dealing with the de Sitter space, one usually considers a specific coordinate system that covers (not necessarily entire) hyperboloid.

The most popular choice of coordinates are those due to Poincaré, without going into details, the metric in this case is as follows:

$$ds^2 = dt^2 - e^{\frac{2t}{R}} dx^2.$$

The latter coincides with the metric of a spatially homogeneous expanding universe but there is no globally defined time-like Killing vector. This metric covers half of the entire de Sitter space — expanding Poincaré patch at Fig. 2. However, we will mostly use static coordinates of the de Sitter space, which are defined as follows:

$$X\left(\frac{t}{R}, \frac{x}{R}\right) = \begin{cases} X^0 = R \sinh\frac{t}{R} \operatorname{sech}\frac{x}{R} \\ X^1 = R \tanh\frac{x}{R} = u \\ X^2 = R \cosh\frac{t}{R} \operatorname{sech}\frac{x}{R} \end{cases}; \quad t \in (-\infty, +\infty);\ x \in (-\infty, +\infty). \tag{2.2}$$

Here and below we set $R = 1$ and $\tanh x \equiv u$. The static coordinate system was introduced as early as 1917 by Willem de Sitter in the course of the famous debate on the relativity of inertia [27]. Of course, it is natural to expect that physical observables do not depend on the choice of the coordinate system. But, in fact, a calculation of some quantities in quantum field theory means to calculate the average of the corresponding operator for a particular state. But the sets of modes of the field in different patches (if, for example, comparing the Poincaré region and the static patch) are not equivalent. Therefore, Fock spaces, generally speaking, do not have to coincide. In other words as one can see from the Fig. 2 one cannot define one universal Cauchy surface for all coordinate systems. However, as will shown later, there are states upon consideration of which it turns out that tree-level propagators can be analytically continued from one region to another.

Figure 2: Penrose diagram for the de Sitter space. The blue shaded part — the static patch(SP), the part with red dots — expanding Poincaré patch(EPP). A line is the Cauchy surface for the field in EPP, B line is the Cauchy surface for the field in SP, C line is the Cauchy surface for the field in the global de Sitter space. Note that if we consider the Cauchy surface in the SP as the Cauchy surface in the EPP, then there are the zones where it becomes light-like — thick red lines. OBS-1 is the observer inside the static patch, while OBS-2 is the observer outside the static patch. Dashed black lines represent the boundary of the causal past for the observers.

The static coordinates cover only the quarter $\{|X^1| < 1\} \cap \{X^2 > |X^0|\}$ of the entire de Sitter manifold (blue shaded region in Fig. 2). The static wedge is itself a globally hyperbolic space time but a Cauchy surface for the wedge is incomplete in relation to the whole de Sitter manifold, being only "one half" of a bona fide Cauchy surface[2]. As one can see from Fig. 2: the Cauchy surface in the static patch does not describe the causal past for the observer-2 at all. On the other hand quantization in the static coordinates has an advantage in comparison with other coordinate systems. From a group theoretical viewpoint the new time coordinate t parametrizes the one-parameter subgroup of the de Sitter group. And as it will be shown later the Hamiltonian operator is time independent. Direct calculation of the induced metric in the static patch gives the following expression:

$$ds^2 = \frac{dt^2 - dx^2}{\cosh x^2}; \tag{2.3}$$

it is time independent and conformal to the flat metric. The static patch is bordered by a bifurcate Killing horizon

$$x \to \pm\infty \quad ; \quad t = \pm x + \text{finite constant};$$

where the metric degenerates. The corresponding Killing vector ∂_t is not time-like when extended outside the static patch. As for the de Sitter invariant combinations of coordinates, the de Sitter invariant scalar product is given by:

$$Z = Z_{12} = X_1 \cdot X_2 = \frac{\cosh(t_1 - t_2) + \sinh x_1 \sinh x_2}{\cosh x_1 \cosh x_2}. \tag{2.4}$$

Note that (2.4) has the following symmetry:

$$Z(t_2 - t_1 + 2\pi i) = Z(t_2 - t_1). \tag{2.5}$$

The geodesic distance L and hyperbolic distance Z are related as follows: $Z = \cosh(L)$ for time-like geodesics, $Z = \cos(L)$ for space-like ones, $Z = 1$ for light-like separations or coincident points. Also note that $Z = -1$ for the antipodal points.

2.2 Canonical Quantization

In this section we outline the canonical quantization of the minimally coupled massive scalar field in the static chart. In this work we consider fields with $m^2 > 1/4$, the reason for this bound will be seen below. We apply standard methods of canonical quantization and look for a complete set of modes by separating the variables, but of course the constructed set of modes will be incomplete when considered w.r.t. the whole de Sitter manifold [29, 30]. Using (2.3) one can find Klein-Gordon equations:

$$\left(\partial_t^2 - \partial_x^2 + \frac{m^2}{\cosh^2 x}\right)\phi(t;x) = 0. \tag{2.6}$$

Let us consider factorized modes which have *positive frequencies* w.r.t. the time coordinate t:

$$\varphi(t;x) = e^{-i\omega t}\,\psi_\omega(u); \quad u = \tanh x. \tag{2.7}$$

[2] In saying this we are *supposing* that the geodesical completion of the wedge is the de Sitter manifold. Would we *suppose* that the geodesical completion be, say, its double covering, the result would change completely. In particular there would be no thermal state at all [28].

$\psi_\omega(u)$ are eigenfunctions of the continuous spectrum of the well-known quantum mechanical scattering problem:

$$\left(-\partial_x^2 + \frac{m^2}{\cosh^2 x}\right)\psi_\omega(u) = \omega^2\,\psi_\omega(u); \quad u = \tanh x; \quad \nu^2 = m^2 - \frac{1}{4}. \tag{2.8}$$

It is clear now why we consider $m^2 > 1/4$ — we want to work with real ν. For any given $\omega \geq 0$ the Ferrers functions $P^{i\omega}_{-\frac{1}{2}+i\nu}(\pm u)$ — also known as Legendre functions on the cut [31] — are two independent solutions of the above equation. At large positive x the wave, which behaves as

$$\psi_\omega(\tanh x) \sim e^{i\omega x}; \quad x \to \infty \tag{2.9}$$

, is purely right moving (at large negative $x \to -\infty$ the wave which behaves as $\psi_\omega(-\tanh x) \sim e^{-i\omega x}$ is purely left moving). The quantized fields should satisfy the canonical commutation relations:

$$\left[\hat\varphi(t;x_1), \hat\varphi(t;x_2)\right] = 0; \quad \left[\hat\varphi(t;x_1), \dot{\hat\varphi}(t;x_2)\right] = i\,\delta(x_1 - x_2). \tag{2.10}$$

But the double degeneracy of the energy level ω points towards the introduction of two pairs of creation and annihilation operators for each level:

$$\left[a_{\omega_1}, a^\dagger_{\omega_2}\right] = \delta(\omega_1 - \omega_2); \quad \left[b_{\omega_1}, b^\dagger_{\omega_2}\right] = \delta(\omega_1 - \omega_2); \quad \left[a_{\omega_1}, b_{\omega_2}\right] = \left[a_{\omega_1}, b^\dagger_{\omega_2}\right] = 0. \tag{2.11}$$

Then the mode expansion of the field operator $\hat\varphi(t;x)$, which obeys (2.6) and (2.10) can then be written as follows:

$$\hat\varphi(t;x) = \int_0^\infty \frac{d\omega}{2\pi}\left[e^{-i\omega t}\left(\psi_\omega(u)\,a_\omega + \psi_\omega(-u)\,b_\omega\right) + e^{i\omega t}\left(\psi^*_\omega(u)\,a^\dagger_\omega + \psi^*_\omega(-u)\,b^\dagger_\omega\right)\right]; \tag{2.12}$$

where

$$\psi_\omega(u) = \sqrt{\sinh(\pi\omega)}\;\Gamma\!\left(\frac{1}{2} + i\nu - i\omega\right)\Gamma\!\left(\frac{1}{2} - i\nu - i\omega\right) P^{i\omega}_{-\frac{1}{2}+i\nu}(u). \tag{2.13}$$

The normalization has been chosen according with the completeness relation (B.1) shown in Appendix A. By normal ordering w.r.t. the Fock vacuum of the a_ω and b_ω operators we get the free Hamiltonian in the standard form

$$:H: = \int_{-\infty}^{+\infty} dx \sqrt{-g}\, :T^0_0: = \int_0^{+\infty} d\omega\, \omega \left(a^\dagger_\omega a_\omega + b^\dagger_\omega b_\omega\right); \tag{2.14}$$

where the orthogonality relation for the associated Legendre functions [32] is used. Note that the range of integration over ω starts from zero (rather than m as for a massive field in flat space). This is because the "mass" term in the action

$$S_m = \int d^2x \sqrt{-g}\, m^2 \varphi^2(t;x);$$

vanishes near the horizon (recall that $\sqrt{-g} = (\cosh x)^{-2}$).

2.3 Thermal two-point functions

The quantum mechanical average over a thermal state of inverse temperature β is given by

$$\langle O \rangle = \frac{\mathrm{Tr}\, \rho O}{\mathrm{Tr}\, \rho}; \qquad \rho \equiv e^{-\beta H}: \tag{2.15}$$

It allows to compute the thermal two-point function at inverse temperature β by assuming the Bose-Einstein distribution of the energy levels

$$\langle a_\omega^\dagger a_{\omega'} \rangle = \langle b_\omega^\dagger b_{\omega'} \rangle = (e^{\beta\omega} - 1)^{-1} \delta(\omega - \omega'): \tag{2.16}$$

Eqs. (2.12) and (2.14) give the following expression for the Wightman function:

$$\begin{aligned} W_\beta(t_1 - t_2; x_1; x_2) = \langle \hat\phi(t_1; x_1)\hat\phi(t_2; x_2)\rangle_\beta &= \int_0^\infty \frac{d\omega}{4\pi^2} \left[\frac{e^{-i\omega(t_1 - t_2)}}{1 - e^{-\beta\omega}} \left(\psi_\omega(u_1)\psi_\omega(u_2) + \right.\right. \\ &\left.\left. + \psi_\omega(-u_1)\psi_\omega(-u_2)\right) + \frac{e^{i\omega(t_1 - t_2)}}{e^{\beta\omega} - 1} \left(\bar\psi_\omega(u_1)\bar\psi_\omega(u_2) + \bar\psi_\omega(-u_1)\bar\psi_\omega(-u_2)\right)\right] = \\ &= \int_{-\infty}^{\infty} e^{-i\omega(t_1 - t_2)} \frac{1 - e^{-2\pi\omega}}{1 - e^{-\beta\omega}} \tilde{P}(\omega; u_1; u_2)\, d\omega; \end{aligned} \tag{2.17}$$

where

$$\tilde{P}(\omega; u_1; u_2) = \frac{e^{\pi\omega}\left[P^{i\omega}_{-\frac12+i\nu}(u_1) P^{-i\omega}_{-\frac12-i\nu}(u_2) + P^{i\omega}_{-\frac12+i\nu}(-u_1) P^{-i\omega}_{-\frac12-i\nu}(-u_2)\right]}{8\cosh\pi(\nu - \omega)\cosh\pi(\nu + \omega)}: \tag{2.18}$$

The states defined by the above two-point functions are mixed. The only pure state is obtained in the limit $\beta \to \infty$.

2.4 Invariant or so-called Bunch-Davis state

As it is stated before cosmologists usually use the spatially flat Poincaré coordinates [16–18], and consider the state, which is invariant with respect to the de Sitter isometry group. In this section, we will rewrite this invariant state as a thermal states in the static patch. In fact, such states is not unique (see [33–35]). However, if we consider only states that correspond to the locally flat propagators, then the only one invariant state left, and this is the so-called the Bunch Davis vacuum [36, 37]. Also well-known Bunch Davis propagator is as follows:

$$W_{BD}(X_1 X_2) = \frac{1}{4\cosh\pi\nu} P_{-\frac12+i\nu}(X_1 X_2) = \frac{1}{4\cosh\pi\nu} P_{-\frac12+i\nu}(Z): \tag{2.19}$$

The argument of this function is the hyperbolic distance between two points on the entire hyperboloid. The restriction to the static patch of the Bunch-Davies state is done with a simple substitution:

$$Z = \frac{\cosh(t_2 - t_1) + \sinh x_1 \sinh x_2}{\cosh x_1 \cosh x_2};$$

and one can see the following symmetry $t_2 - t_1 \to t_2 - t_1 + 2\pi i$, which signals, that pure state in Poincaré coordinates is a mixed state in static patch, moreover this mixed state is thermal one with $\beta = 2\pi$. Indeed if we consider the case $\beta = 2\pi$ the two-point function

(2.17) is the de Sitter invariant and coincides with the restriction to the static patch of the Bunch-Davies two-point function (2.19):

$$W_2 \ (t_1 \quad t_2;\ x_1; x_2) = W_{BD}(Z) = \frac{1}{4\cosh} P_{\frac{1}{2}+i}(Z): \qquad (2.20)$$

On the other hand for arbitrary the two point function $W\ (t_1 \quad t_2; x_1; x_2)$ and its permuted function do not respect the de Sitter isometry because their periodicity thermal property in imaginary time $t\ ! \quad t + i$ is incompatible with the geometry of the global de Sitter manifold, which has the following symmetry (2.5), the only exception being $= 2$. The others invariant states (so-called states) in terms of the static patch states are disscussed in [24].

2.5 Light-like separation

For light like separations the propagators should behave as in Minkowski space. In the Bunch-Davies invariant case this comes immediately from Eq. (2.20):

$$W_{BD}(Z \qquad 1) \qquad \frac{1}{4}\log(1+Z) \qquad \frac{1}{4}\log\left[t^2 \quad (x_1 \quad x_2)^2\right]: \qquad (2.21)$$

For arbitrary at light-like separation large values of 's dominate in the integral (2.17). For large ! we may approximate $P^{i!}_{\frac{1}{2}+i}$ (tanh x_1) $e^{i!x_1} = (1 \quad i!\ )$ and get the leading term

$$W\ (t; x_1; x_2) \qquad \int_1 \frac{d!}{2}\frac{e^{i!t}}{2!} \quad e^{i!\ (x_1 \quad x_2)} + e^{i!\ (x_1 \quad x_2)} \qquad \frac{1}{4}\log\left[t^2 \quad (x_2 \quad x_1)^2\right]:$$

The cuto in this integral is order of $R^{\ 1}$ the radius of the de Sitter universe, which we set to one. The approximation works for $j!\,j$ much larger than m and $R^{\ 1}$. The dependence on the temperature is lost in this high energy limit: only the Hadamard term survives.

2.6 Anomalous singularities at the horizon

When the temperature is an integer multiple of the Gibbons-Hawking temperature, i.e. when $= 2 = N$, we may use Eq. (2.17) to derive another representation of the two-point function as a nite sum of Legendre functions, compare to the in nite Matsubara-type series:

$$W\ (X_1; X_2) \ = \sum_{n=0} W_1\ (t_1 \quad in\ ; x\ _1; t_2; x_2) + \sum_{n=1} W_1\ (t_2; x_2; t_1 + in\ ; x\ _1):$$

Where W_1 is Wightman function with zero temperature. So for the cases $= 2 = N$ (see also [35, 38]):

$$W_{\frac{2}{N}}(t_1 \quad t_2;\ x_1; x_2) \ = \int_1^{+1} e^{\ i!\ (t_1 \quad t_2)}\frac{1 \quad e^{\ 2\,!}}{1 \quad e^{\ \frac{2\,!}{N}}} P_{!;}\ (u_1; u_2)\ d! \ =$$

$$= \frac{1}{4\cosh} P_{\frac{1}{2}+i}\ \ Z\ \ t_1 \quad t_2 \quad i\ ; x_1; x_2 \quad +$$

$$+ \frac{1}{4\cosh} \sum_{n=1}^{N-1} P_{\frac{1}{2}+i} \left(Z \quad t_1 \quad t_2 \quad i\frac{2n}{N}; x_1; x_2 \right) : \quad (2.22)$$

The rst term on the RHS is exactly the Bunch Davies de Sitter invariant Wightman function; this is singular at $Z = 1$. The extra terms become singular only when the two points approach either the left or the right horizon:

$$X_1 = X(+ c_1;); \quad X_2 = X(+ c_2; +): \quad (2.23)$$

In the limit $! \ 1$ the above events belong to the horizons. Then:

$$Z \quad c_1 \quad c_2 \quad i\frac{2n}{N}; ; \quad + \quad = \frac{\cosh c_1 \quad c_2 \quad i\frac{2n}{N} + \sinh \quad \sinh(+)}{\cosh \ \cosh(+)} \ ! \quad 1:$$

Then taking in Eq. (2.22) the horizon limit gives $W_{\frac{2}{N}}(! \ 1 \ ) \quad N\,W_{BD}(! \ 1 \ ):$ For generic , the limit $! \ 1$ may be obtained by performing manipulations similar to those which led to (??):

$$W(! 1) \quad \frac{1}{2}\int_{1}^{+1} d! \frac{1}{e^{(! + i0)} \quad 1 \ \sinh \ (! + i0)} e^{2i!} :$$

Due to presence of the double pole at $= \ i0$ the answer is as follows:

$$W(! 1) \quad \frac{2}{\ } \_ \quad \frac{2}{\ } W_{BD}(! 1):$$

A remarkable fact is the following: for light like separations inside the static patch the dominant contribution to the propagator comes from large $!$'s; on the contrary, at the horizon small $!$'s provide the leading contribution. This is because the horizon is the boundary of the patch; the main contribution comes from the infrared rather than ultra-violet frequencies. The infrared limit of the propagator depends on the temperature, but is independent of the mass m.

2.7 Dependence on coordinate choice

The peculiarity of the propagator appears on the horizon, but the position of the horizon depends on the position of the static patch, which we have chosen in (2.2). The purpose of this section is to clarify that the peculiarity is associated with the speci c choice of the state, rather than the speci c choice of the coordinate system. Let us consider the $=$ case. Then:

$$W(X_1; X_2) = \frac{1}{4\cosh} P_{\frac{1}{2}+i} \left(\frac{\cosh(t_2 \quad t_1) + \sinh(x_1)\sinh(x_2)}{\cosh(x_1)\cosh(x_2)} \right) +$$
$$+ \frac{1}{4\cosh} P_{\frac{1}{2}+i} \left(\frac{\cosh(t_2 \quad t_1) + \sinh(x_1)\sinh(x_2)}{\cosh(x_1)\cosh(x_2)} \right) : \quad (2.24)$$

Inverse coordinate transformation inside the static patch is:

$$x = \operatorname{Arctanh} X^1 ; \qquad t = \operatorname{Arcsinh} \frac{X^0}{\sqrt{1 \quad (X^1)^2}} ;$$

then (2.24) can be rewritten as follows:

$$W_\beta(X_1;X_2) = \frac{1}{4\cosh\pi\nu} P_{-\frac{1}{2}+i\nu}\left(X_1^0X_2^0 - X_1^1X_2^1 - X_1^2X_2^2\right) + \frac{1}{4\cosh\pi\nu} P_{-\frac{1}{2}+i\nu}\left(-X_1^0X_2^0 - X_1^1X_2^1 + X_1^2X_2^2\right).$$

In fact, the argument of Legendre function in the second term can be obtained from the argument in the first term. For that one of the points is reflected with respect to the axes X^0 and X^2. This result resembles the method of image charges from classical electrodynamics, and actually the Wightman function (2.24) is the solution of the equations of motion with specific boundary conditions, thus the peculiarity of the propagator on the horizon is a property of the particular state, but not of the coordinate system.

2.8 Different temperatures for left and right movers

In [24] we constructed general time translation invariant states

$$\langle a^\dagger_\omega a_{\omega'}\rangle = \delta(\omega-\omega')\frac{1}{e^{\beta_R(\omega)\omega}-1} \quad \text{and} \quad \langle b^\dagger_\omega b_{\omega'}\rangle = \delta(\omega-\omega')\frac{1}{e^{\beta_L(\omega)\omega}-1}. \tag{2.25}$$

We gave in particular a full treatment for states of arbitrary global (inverse) temperature

$$\beta_L(\omega) = \beta_R(\omega) = \beta; \tag{2.26}$$

and provided new integral representations for their correlation functions. Taking inspiration from the consideration of Unruh state for black holes, we enlarge that study and consider different global temperatures for the left and the right moving modes:

$$\beta_L(\omega) = \beta_L; \qquad \beta_R(\omega) = \beta_R. \tag{2.27}$$

The Wightman function is the sum of two contributions

$$W_{\beta_L\beta_R}(X_1;X_2) = W_{L,\beta_L}(X_1;X_2) + W_{R,\beta_R}(X_1;X_2); \tag{2.28}$$

where

$$W_{L,\beta}(X_1;X_2) = \int_0^\infty \frac{d\omega}{4\pi^2}\left[e^{-i\omega(t_1-t_2)}\frac{\psi_\omega(-x_1)\psi^*_\omega(-x_2)}{1-e^{-\beta\omega}} + e^{i\omega(t_1-t_2)}\frac{\psi^*_\omega(-x_1)\psi_\omega(-x_2)}{e^{\beta\omega}-1}\right] \tag{2.29}$$

$$W_{R,\beta}(X_1;X_2) = \int_0^\infty \frac{d\omega}{4\pi^2}\left[e^{-i\omega(t_1-t_2)}\frac{\psi_\omega(x_1)\psi^*_\omega(x_2)}{1-e^{-\beta\omega}} + e^{i\omega(t_1-t_2)}\frac{\psi^*_\omega(x_1)\psi_\omega(x_2)}{e^{\beta\omega}-1}\right] \tag{2.30}$$

The formal proof of the KMS periodicity property goes as follows:

$$W_{R,\beta}(X_2(t_2;x_2);X_1(t_1;x_1)) = \frac{1}{4\pi^2}\sum_{n=0}^{\infty}\int_0^\infty e^{-i\omega(t_2-t_1-in\beta)}\psi_\omega(x_2)\psi^*_\omega(x_1)d\omega + \frac{1}{4\pi^2}\sum_{n=1}^{\infty}\int_0^\infty e^{i\omega(t_2-t_1+in\beta)}\psi^*_\omega(x_2)\psi_\omega(x_1)d\omega = W_{R,\beta}(X_1(t_1-i\beta;x_1);X_2(t_2;x_2)) \tag{2.31}$$

There holds the exchange symmetry

$$W_{R,\beta_R}(X_1(t_1;x_1);X_2(t_2;x_2)) = W_{L,\beta_R}(X_1(t_1;-x_1);X_2(t_2;-x_2)). \tag{2.32}$$

When $_L = \ _R = 2$ the Wightman function (2.28) respects the de Sitter isometry [24, 39 44], i.e. it is a function of the complex de Sitter invariant variable

$$Z = \frac{\cosh(t_2 \quad t_1) + \sinh x_1 \sinh x_2}{\cosh x_1 \cosh x_2};$$

with the locality cut on the negative reals Z 1. Let us consider now the behavior at the horizon of the propagator (2.28). Points of the right (left) future horizon are obtained in the following limit

$$\lim_{! \ +1} X(\ ;\ (\qquad)) = \lim_{! \ +1} @ \begin{matrix} 0 \\ \mathrm{sech}(\quad)\sinh \\ \tanh(\quad) \\ \mathrm{sech}(\quad)\cosh \end{matrix} \begin{matrix} 1 \\ \\ A \\ \\ \end{matrix} = @ \begin{matrix} 0 \\ e \\ 1 \\ e \end{matrix} \begin{matrix} 1 \\ \\ A \\ \\ \end{matrix} : \qquad (2.33)$$

Points of the left (right) past horizon are obtained in the limit $! \ 1$ of the above expression. In all cases the interval between two points having the same nite coordinate is spacelike:

$$L_{12} = \frac{2(\cosh(\ _1 \quad _2) \quad 1)}{\cosh(\quad _1)\cosh(\quad _2)} < 0; \qquad (2.34)$$

becoming light like only in the limit $! \ 1$.

Using the asymptotics of the modes:

$$P^{i!}_{\frac{1}{2}+i}(\tanh x) \quad_{x!1} \quad \frac{e^{i!x}}{(1 \quad i!\ )}; \qquad (2.35)$$

$$P^{i!}_{\frac{1}{2}+i}(\ \tanh x) \quad_{x!1} \quad \frac{i! \quad e^{\ i!x}}{\frac{1}{2}+i \quad i! \quad \frac{1}{2} \quad i \quad i!} + \frac{\cosh(\ ) \quad i! \quad e^{i!x}}{} ; \qquad (2.36)$$

and the eq. (A.1), one can obtain the behaviour of $W_{R;\ R}$ and $W_{L;\ L}$ separately at e.g. the right side of the horizon. As we can see from Eq. (2.35), in this region $W_{R;\ R}$ depends only on the di erence $x_1 \quad x_2$, which does not grow when both points are taken to the same side of the horizon. It means, that this contribution to the Wightman function is regular near the right side of the horizon. At the same time, in the same region $W_{L;\ L}$ depends on the both $x_1 \quad x_2$ and $x_1 + x_2$, as we can see from Eq. (2.36). The latter sum is in nitely growing near the horizon. As the result, using (A.1) one obtains that:

$$W_{\ L\ R}\ X(\ ;\ _1\quad);X(\ ;\ _2\quad)$$

$$W_{L;\ L}\ X(\ ;\ _1\quad);X(\ ;\ _2\quad) \quad \frac{1}{_L}; \qquad ! \ +1 : \quad (2.37)$$

Behavior near the left horizon can be also found, and follows from the relation:

$$W_{\ L\ R}(X(t_1;x_1);X(t_2;x_2)) = W_{\ R\ L}(X(t_1;\ x_1);X(t_2;\ x_2)) :$$

Namely parity $x \ ! \quad x$ plus rearrangement of temperatures $_L \ \$ \ _R$ leave the two-point-function invariant. As a result, for $! \ 1$ we obtain

$$W_{\ L\ R}\ X(\ ;\ _1\quad);X(\ ;\ _2\quad) \quad W_{R;\ R}\ X(\ ;\ _1\quad);X(\ ;\ _2\quad) \quad \frac{1}{_R} j \ j: \qquad (2.38)$$

Thus light like singularity at the horizons depends on the state of the theory. In particular, at the right horizon it depends only on $_R$, while at the left horizon it depends on $_L$. This shows that such a peculiar behavior of propagators is present due to the interplay between the waves that are falling down and re ected from the $m^2 = \cosh^2 x$ potential.

2.9 General dimension

The $(D+1)$-embedding coordinates and the invariant scalar product for the D dimensional static patch are given by

$$X_0 = \sinh(t)\,\mathrm{sech}(x); \quad X_i = \tanh(x)\, y_i; \quad X_D = \cosh(t)\,\mathrm{sech}(x); \quad y_i y_i = 1; \tag{2.39}$$

$$Z = \quad X_1 X_2 = \frac{\cosh(t_2 \quad t_1) + y_1 \quad y_2 \sinh x_1 \sinh x_2}{\cosh x_1 \cosh x_2}: \tag{2.40}$$

The Bunch-Davies Wightman function [36, 37, 39, 42, 43, 45 47] corresponding to the inverse temperature $_L = \quad _R = 2$ [35, 39, 41 43] is given by

$$W_2 (Z) = \frac{\left(\frac{D-1}{2} + i\right) \quad \left(\frac{D-1}{2} \quad i\right)}{2(2\ )^{\frac{D}{2}}} (Z^2 \quad 1)^{\frac{D-2}{4}} P^{\frac{D-2}{2}}_{\frac{1}{2}+i} (Z); \tag{2.41}$$

where $= \sqrt{m^2 \quad (D \quad 1)^2 = 4}$. It has the standard Hadamard singularity near $Z = \quad 1$.

Points of the future and past horizons are attained in the following limits

$$\lim_{\ \to 1} X(\ ;\ (\quad)) = \lim_{\ \to 1} \begin{pmatrix} \mathrm{sech}(\quad)\sinh \\ \tanh(\quad) y \\ \mathrm{sech}(\quad)\cosh \end{pmatrix} = \begin{pmatrix} e \\ y \\ e \end{pmatrix} \tag{2.42}$$

Two events on the horizons are spacelike separated unless $y_1 = \ y_2$. As in Eq. (3.17) for $= \frac{2}{N}$ in the horizon limit one gets:

$$W_{\frac{2}{N}}(\ \ !\ \ +1\ ) \quad N\,W_2(\ \ !\ \ +1\ ) \quad N \frac{\frac{D-2}{2}}{2^{2+(D\ \ 2)\frac{3}{2}} \quad \frac{D}{2}} e^{(D\ \ 2)} : \tag{2.43}$$

As in Rindler space the singularity of the propagator on the horizon depends on the temperature.

2.10 Stress-energy tensor in the 2D de Sitter space

Let us consider now the expectation value of the stress energy tensor in the 2D theory in the static patch. We use point-splitting regularization method. It is discussed in details in Appendix C. To set up notations let us discuss first the de Sitter invariant case $= 2$. When the two arguments of the Wightman function are taken very close to each other, one has that

$$W_2 (X_+; X\ ) \quad \frac{1}{4} \quad H_{\frac{1}{2}+i} + H_{\frac{1}{2}\ \ i} + \log \frac{(V \quad V_+)(U_+ \quad U\ )}{4\cosh^2(V \quad U)} \quad ; \tag{2.44}$$

where $H_{\frac{1}{2}+i} = \quad \frac{1}{2} + i \quad + \ _e$ are the harmonic numbers; the definitions of X , V and U can be found in appendix B. Since

$$@_{V_+} @_V\ W_2 (Z) \quad \frac{1}{4\ (V_+ \quad V\ )^2} + \frac{1}{48}; \quad \text{and} \quad @_{U_+} @_U\ W_2 (Z) \quad \frac{1}{4\ (U_+ \quad U\ )^2} + \frac{1}{48}: \tag{2.45}$$

the covariant point splitting regularization gives

$$\langle T_{UV} \rangle_2 = \frac{m^2}{8\ \cosh^2(\frac{V\ \ U}{2})} \quad \frac{1}{2} + i \quad + \quad \frac{1}{2} \quad i \quad + 2\ _e + \log\ ^2 t\ t \quad ; \tag{2.46}$$

$$\langle T_{UU} \rangle_2 = \quad \frac{1}{4\ ^2(t\ \ t\ )} + \frac{R}{24} \quad \frac{t_U t_U}{t\ \ t}; \tag{2.47}$$

$$\langle T_{VV} \rangle_2 = \quad \frac{1}{4\ ^2(t\ \ t\ )} + \frac{R}{24} \quad \frac{t_V t_V}{t\ \ t}: \tag{2.48}$$

After regularization we obtain the well known answer [36], see also [48, 49].

$$h T \quad :i_2 = \frac{1}{4}m^2 \quad \frac{1}{2} + i \quad + \quad \frac{1}{2} \quad i \quad + 2 \quad _e \; g \quad + \frac{R}{48} g \qquad (2.49)$$

The expectation value of the stress energy tensor with two temperatures $_L$ and $_R$ can be obtained starting from Eq. (2.28). The most interesting case in this situation is the near horizon limit. For instance, close to right horizon a lengthy but not di cult calculation gives

$$@_+ @ \; W_{L \; R}(X^+; X \;) \quad \int_1^1 \frac{d!}{4} \frac{!}{e^{L!} \quad 1} e^{i!\,(V^+ \quad V\;)} = \frac{}{12\;{}_L^2} \quad \frac{1}{4} \frac{1}{(V^+ \quad V\;)^2}; \qquad (2.50)$$

and

$$@_+ @ \; W_{L \; R}(X^+; X \;)$$
$$\int_1^1 \frac{d!}{4} \frac{! \;\sinh^2 \;! \; e^{i!\,(U^+ \quad U\;)}}{\cosh\;(! \qquad)\cosh\;(! \; + \quad)} \quad \frac{1}{e^{R!} \quad 1} + \frac{\cosh^2}{\sinh^2 \;!} \frac{1}{e^{L} \quad 1} \; : \qquad (2.51)$$

The above expressions simplify when the temperatures of the left and right movers coincide: $_R = \; _L = \;$. Then the regularized stress-energy tensor in the near horizon limit takes the form:

$$h T \quad :i \qquad + \frac{R}{48} g \;; \qquad (2.52)$$

where

$$_{UU} = \frac{1}{12} C^{1=2} @_U^2 C^{1=2} + \frac{}{12\;{}^2} = \frac{}{12} \quad \frac{1}{{}^2} \quad \frac{1}{(2\;)^2} \;;$$
$$_{VV} = \frac{1}{12} C^{1=2} @_V^2 C^{1=2} + \frac{}{12\;{}^2} = \frac{}{12} \quad \frac{1}{{}^2} \quad \frac{1}{(2\;)^2} \;;$$
$$_{UV} = \; _{VU} = 0:$$

the de Sitter covariance is recovered only when $= 2$. Note that for generic values of $_L$ and $_R$ the expectation valur $h T \quad :i$ is singular in the free falling reference frame.

3 Quantum elds in the Rindler space-time

We have seen that in the de Sitter space anomalous divergence arise on the horizon. So in this section we will do similar calculations in the Rindler space-time, because as it is stated before, the metric in the Rindler space-time is an approximation of the metric in the static de Sitter space near the horizons. This section mainly contains a recapitulation of known facts. However some of them are new. Also see [25].

3.1 Geometry, modes and Wightman function

The Rindler space covers right quarter of the Minkowski space-time (see Fig. 3):

$$\begin{matrix} t \\ x \end{matrix} = \begin{matrix} e \; \sinh(\;) \\ e \; \cosh(\;) \end{matrix} \quad : \qquad (3.1)$$

Figure 3: Penrose diagram of the Rindler space. The Rindler space is bordered by a Killing horizon.

Rindler space is invariant under one-parameter group of boosts:

$$\begin{pmatrix} \cosh(\ ) & \sinh(\ ) \\ \sinh(\ ) & \cosh(\ ) \end{pmatrix} \begin{pmatrix} e\ \sinh(\ ) \\ e\ \cosh(\ ) \end{pmatrix} = \begin{pmatrix} e\ \sinh(\ +\ ) \\ e\ \cosh(\ +\ ) \end{pmatrix} : \qquad (3.2)$$

Here is time-like coordinate, is the space-like coordinate. The proper acceleration is choosen to be one. As it is stated before, for real[3] values of and the Rindler coordinates (3.1) cover only the right wedge; this is causally disconnected from the left wedge. In the above coordinates the metric has the following form:

$$ds^2 = e^2\ \ d\ ^2\ \ d\ ^2 : \qquad (3.3)$$

The half-lines $t = \ x;\ x > 0$ are the past and the future horizons. Geodesic distance between two points in the Right wedge is given by

$$L_{12} = (t_2\ \ t_1)^2\ \ (x_2\ \ x_1)^2 = 2e^{(\ _1 +\ _2)} \cosh(\ _2\ \ _1)\ \ e^{2\ _2}\ \ e^{2\ _1} : \qquad (3.4)$$

Also please note the obvious symmetry of L_{12} under the exchange

$$_1\ !\ \ _2 : \qquad (3.5)$$

Lorentz transformations of the wedge correspond to (time) translations in the variable: $!\ \ +$, dilatations in Minkowski space $X\ !\ \ e\ X$ correspond to the shift $!\ \ +$.

As regards the light cone variables

$$u = t\ \ x = \ e^{(\ \ )} = \ e^{U} ; \qquad v = t + x = e^{(\ +\ )} = e^{V} ; \qquad (3.6)$$

[3] Analytical continuation to the others wedges is an important issue, which, however, is beyond the topic under discussion

they are transformed as follows:

$$\begin{aligned} &u \to e^{-\lambda} u; \quad \bar{u} \to e^{\lambda} \bar{u}; \quad U \to U+\lambda; \quad \bar{U} \to \bar{U}-\lambda; \\ &v \to e^{-\lambda} v; \quad \bar{v} \to e^{\lambda} \bar{v}; \quad V \to V+\lambda; \quad \bar{V} \to \bar{V}+\lambda. \end{aligned} \tag{3.7}$$

The Rindler space is geodesically incomplete. Of course a Cauchy surface in it, say $\eta = 0$, is not a Cauchy surface for the whole Minkowski spacetime. As a consequence the modes constructed by canonical quantization in the Rindler wedge do not constitute a basis for the whole Minkowski space time. It is well-known that to obtain general Hilbert space representations of the fields, one needs to construct also the modes defined in the left wedge [50]. A less known but powerful alternative is to resort to the theory of generalized Bogoliubov transformations[4] which makes use only of the modes of the right wedge [29, 30].

The Klein Gordon equation for a massive scalar field in two-dimensions is as follows:

$$\left(\partial_\eta^2 - \partial_\xi^2 + e^{2\xi} m^2\right) \varphi(\eta;\xi) = 0. \tag{3.8}$$

By separating the variables one gets a Schrodinger eigenvalue (textbook) problem in an exponential potential $V(\xi) = m^2 e^{2\xi}$. Because the potential increases infinitely as $\xi \to +\infty$ we have two types of solutions of the eqn. (3.8): the first increases at infinity while the second one decreases. Normalizable modes (which exponentially decay as $\xi \to +\infty$) are proportional to Macdonald functions $K_{i\omega}(m e^{\xi})$. Note, that these modes are linear combinations of left-moving and right-moving waves. The canonical field operator is as follows

$$\hat{\varphi}(\eta;\xi) = \frac{1}{\pi}\int_0^{+\infty} \left(e^{-i\omega\eta}\, \hat{b}_\omega + e^{i\omega\eta}\, \hat{b}_\omega^\dagger \right) K_{i\omega}\left(m e^{\xi}\right) \sqrt{\sinh \pi\omega}\, d\omega, \tag{3.9}$$

where the creation and annihilation operators obey the standard commutation relations:

$$[\hat{b}_\omega ; \hat{b}^\dagger_{\omega'}] = \delta(\omega - \omega'); \qquad [\hat{b}_\omega ; \hat{b}_{\omega'}] = 0.$$

The so-called Fulling vacuum [13, 51] is identified by the condition

$$\hat{b}_\omega |0_R\rangle = 0; \quad \omega \geq 0. \tag{3.10}$$

It is a pure state and the corresponding two-point function is given by

$$W_\infty(X_1;X_2) = \langle 0_R|\hat{\varphi}(X_1)\hat{\varphi}(X_2)|0_R\rangle = \frac{1}{\pi^2}\int_0^\infty e^{-i\omega(\eta_1-\eta_2)} K_{i\omega}(m e^{\xi_1}) K_{i\omega}(m e^{\xi_2}) \sinh \pi\omega\, d\omega. \tag{3.11}$$

But as one can see, this two-point function is not invariant under Poincaré group of the entire Minkowski space. It is well-know that the invariant state is thermal one in terms of modes in Right wedge. The thermal equilibrium average of an operator $\hat{O}$ at temperature $T = \beta^{-1}$ is defined in quantum mechanics as follows:

$$\langle \hat{O} \rangle = \frac{\mathrm{Tr}\, e^{-\beta \hat{H}} \hat{O}}{\mathrm{Tr}\, e^{-\beta \hat{H}}}, \tag{3.12}$$

where $\hat{H}$ is the Hamiltonian of the system. In terms of (3.9) it has the following form:

$$:\hat{H}: = \int_0^\infty d\omega\, \omega\, \hat{b}^\dagger_\omega \hat{b}_\omega . \tag{3.13}$$

---

[4] Starting from pure states generalized Bogoliubov transformations may produce mixed states while standard Bogoliubov transformations cannot.

Note that the spectrum of the eld starts from zero and not from mass, as that is the case in Minkowski space-time. In the case under consideration thermal two point function is given by the following form:

$$W (X_1(_1; _1); X_2(_2; _2)) = \frac{1}{2} \int_1^{+1} \frac{e^{i! (_1 \quad _2)}}{e^{!} \quad 1} K_{i!} (me^{1}) K_{i!} (me^{2}) \sinh ! d!: \qquad (3.14)$$

The two-point function (3.14) is time-translation invariant (and therefore it provides an equilibrium state at least at tree-level). When $= 2$ an explicit calculation of the integral shows that the Wightman function in Eqn. (3.14) can be extended to the whole complex Minkowski spacetime (minus the causal cut) and it is actually Poincaré invariant [14, 29, 30]:

$$W_2 (X_1; X_2) = \frac{1}{2} K_0 \; m \sqrt{\quad L} \; : \qquad (3.15)$$

At close points (when $mL ! 0$) it has the standard ultraviolet (Hadamard behaviour) divergence with the correct coe cient $1=4$:

$$\frac{1}{2} K_0 \; m \sqrt{\quad L} \qquad \frac{1}{4} \log(m^2 L): \qquad (3.16)$$

Inside the Rindler wedge, the main contributions to the integral (3.14) for light like separations come from high energies $me^{1;2}$ ($_{1;2}$ xed). This is the ultraviolet e ect and the divergence does not depend on the temperature. This is true for any However, when $6 2$ there are extra (anomalous) singularities at the horizon the boundary of the wedge, which of course is also light like. We will show this now.

When the temperature is an integer multiple of the canonical one ($= \frac{2}{n}$) a simple formula is available [24, 35]:

$$W_{\frac{2}{N}} (X_1; X_2) = \sum_{k=0}^{1} W_2 \quad X_1 \quad _1 \quad \frac{2 i k}{n}; \; _1 \quad ; X_2(_2; _2) \quad : \qquad (3.17)$$

Let us consider the simplest case $=$:

$$W \;=\; \frac{1}{2} K_0 \; m \sqrt{e^{2 _1} + e^{2 _2} \quad 2e^{_1 + _2} \cosh} \;+\; \frac{1}{2} K_0 \; m \sqrt{e^{2 _1} + e^{2 _2} + 2e^{_1 + _2} \cosh} \quad : \qquad (3.18)$$

Points of the horizons may be attained as follows:

$$\lim_{! 1} X (; \quad) = \lim_{!1} \begin{matrix} e & \sinh \\ e & \cosh \end{matrix} \;=\; \frac{1}{2} \begin{matrix} e \\ e \end{matrix} \quad : \qquad (3.19)$$

The interval between two points having the same coordinate is spacelike; for instance

$$L_{12} = (X_1(; \; _1 \quad) \quad X_2(; \; _2 \quad))^2 = \quad e^{2} (e^{1} \quad e^{2})^2 < 0; \qquad (3.20)$$

furthermore

$$(X_1(\quad i ; \; _1 \quad) \quad X_2(; \; _2 \quad))^2 = \quad e^{2} (e^{1} + e^{2})^2 = L_{12} + 4e^{2} e^{1+ 2}: \qquad (3.21)$$

The rst term in (3.18) is singular for any two light-like separated points in the Rindler wedge. When the two points are both approaching either the future or the past horizon also the second term diverges, and it does exactly as the rst term; when $! +1$:

$$W \quad X (; \; _1 \quad); X (; \; _2 \quad) \qquad \frac{2}{4} \log(m^2 L_{12}); \quad \text{as} \quad ! \quad +1 : \qquad (3.22)$$

Similarly for $\beta = \frac{2\pi}{N}$ and $\xi \to +\infty$.

$$W_{\frac{2\pi}{N}}\left(X(\eta;\xi_1);X(\eta;\xi_2)\right) \approx -\frac{N}{4\pi}\log(m^2 L_{12}) = -\frac{1}{2\beta}\log(m^2 L_{12}). \quad (3.23)$$

In the horizon limit (3.19) the dominant contribution to the integral (3.14) comes from the infrared region $\omega \to 0$. Using appendix A and asymptotic form of the modes near horizon one can show that (3.23) remains true for general β. The calculation is similar to the one preformed in [24]. Such a dependence of the coefficient of the singularity at light like separation (at the horizon) implies that the thermal state cannot be continued to the entire Minkowski space time.

It is possible to introduce more general time translation invariant (at tree level) states by letting the temperature depend on the energy:

$$W(X_1;X_2) = \frac{1}{\pi^2}\int_0^{\infty}\left[\frac{e^{-i\omega(\eta_1-\eta_2)}}{1-e^{-\beta(\omega)\omega}} + \frac{e^{i\omega(\eta_1-\eta_2)}}{e^{\beta(\omega)\omega}-1}\right]K_{i\omega}(me^{\xi_1})K_{i\omega}(me^{\xi_2})\sinh\pi\omega\, d\omega. \quad (3.24)$$

These states also respect the exchange symmetry (3.5).

Such a behaviour at the horizon is quite unexpected result, because massive term is included into the action as:

$$S_m = \int d\eta\, d\xi \sqrt{-g}\, m^2\varphi^2(\eta,\xi) = \int d\eta\, d\xi\, e^{2\xi} m^2 \varphi^2(\eta,\xi),$$

and degenerates at the horizon exponentially ($\sqrt{-g} = e^{2\xi} \to 0$). So it seems that locally near horizon the field should behaves as the massless one.

3.2 General dimension

The discussed anomalous singularity of propagators is not a property of just two dimensional case. For the field in general dimension we obtained the same result. Consider $D-2$ extra flat transverse spatial dimensions $\vec z$. Then the Rindler metric is as follows:

$$ds_D^2 = e^{2\xi}\left(d\eta^2 - d\xi^2\right) - d\vec z^2. \quad (3.25)$$

The modes can be represented as $\varphi_D(\eta;\xi;\vec z) = e^{i\vec k_\perp \vec z}\varphi_{\vec k_\perp}(\eta;\xi)$ where $\varphi_{\vec k_\perp}(\eta;\xi)$ obeys Eq. (3.8) with the effective mass $m^2 + k_\perp^2$. Therefore the field operator can be expanded as

$$\hat\varphi(\eta;\xi;\vec z) = \int_{-\infty}^{+\infty}\frac{d^{D-2}k_\perp}{(2\pi)^{\frac{d}{2}}}\int_0^{+\infty}\frac{d\omega}{\pi}\sqrt{\sinh\pi\omega}\left(e^{-i\omega\eta+i\vec k_\perp\vec z}\,\hat b_{\omega,\vec k_\perp} + e^{i\omega\eta-i\vec k_\perp\vec z}\,\hat b^\dagger_{\omega,\vec k_\perp}\right)K_{i\omega}\left(\sqrt{m^2+k^2}\,e^{\xi}\right). \quad (3.26)$$

The Wightman function at temperature β is as follows:

$$W^D_\beta(X_1;X_2) = \int_{-\infty}^{+\infty}\frac{d^{D-2}k_\perp}{(2\pi)^d}\int_{-\infty}^{+\infty}\frac{d\omega}{\pi^2}\frac{\sinh(\pi\omega)}{1-e^{-\beta\omega}}e^{-i\omega(\eta_1-\eta_2)}e^{i\vec k_\perp(\vec z_1-\vec z_2)}K_{i\omega}\left(\sqrt{m^2+k^2}\,e^{\xi_1}\right)K_{i\omega}\left(\sqrt{m^2+k^2}\,e^{\xi_2}\right) \quad (3.27)$$

Poincaré invariance is respected for $\beta = 2\pi$ [30]:

$$\int_{-\infty}^{+\infty}\frac{d^dk}{(2\ )^d}\int_{-\infty}^{+\infty}\frac{d!}{2\ ^2}e^{!}\ e^{\ i!\ (\ _1\ \ _2)}e^{i\bar{k}_?(\bar{z}_1\ \ \bar{z}_2)}K_{i!}\ \ \sqrt{m^2+\bar{k}_?^2}\,e^{\ _1}\ \ K_{i!}\ \ \sqrt{m^2+\bar{k}_?^2}\,e^{\ _2}\ =$$

$$=\int_{-\infty}^{\infty}\frac{d^{D\ \ 2}k_?}{2\ \ (2\ )^{\frac{D\ \ 2}{2}}}\ e^{i\bar{k}_?\ \ \bar{z}}K_0\ \ \sqrt{m^2+\bar{k}_?}\ \ \sqrt{\ L_2}\ \ =$$

$$=\frac{1}{2}\ \ \frac{\sqrt{L_2+j\ \ \bar{z}j^2}}{m}^{\ \frac{D\ \ 2}{2}}\ K_{\frac{D\ \ 2}{2}}\ \ m\sqrt{L_2+j\ \ \bar{z}j^2}\ ;\quad (3.28)$$

where $\bar{z}\ \ \ \bar{z}_1\ \ \ \bar{z}_2$ and L_2 is two dimensional Rindler space part of Geodesic distance and coincide with (3.4):

$$L_2=2e^{(\ _1+\ _2)}\cosh(\ _2\ \ \ _1)\ \ \ e^{2\ _2}\ \ \ e^{2\ _1}:\qquad (3.29)$$

Then geodetic distance between two points is de ned as follows:

$$L_{12}^D=(\ \ t)^2\ \ \ (\ \ x)^2\ \ \ (\ \ \bar{z})^2=L_2\ \ \ (\ \ \bar{z})^2:\qquad (3.30)$$

The anomalous divergence on the horizon for generic $\ \ 6\ 2$ goes precisely as in the previous section. Let us consider the horizon limit $_1\ \ \ _2\ !\ \ 1$, then $L_2\ !\ \ 0$. Also let us consder the following limig: $L_2\ \ j\ \ \ \bar{z}j^2\ \ \ \ m^{\ \ 2}$. So near the horizon for the $\ =\frac{2}{N}$:

$$W\ \ (X_1;X_2)\ \ \ \ N\frac{\frac{D\ \ 2}{2}}{4}\ \frac{1}{j\ \ \ \bar{z}j^{D\ \ 2}}:\qquad (3.31)$$

(3.31) has a standart peculiarity in the limit $j\ \ \bar{z}j\ !\ \ 0$, but with wrong coe cient N. Also (3.31) can be obtained approximately from the mode expansion. Let us put $_1=\ _2=\ \ \ !\ \ 1$, then:

$$K_{i!}\ \ \sqrt{m^2+k_?^2e}\ \ \ \ \ \sqrt{\frac{}{!\ \sinh\ !}}\sin\ !\ \log\ \sqrt{m^2+k_?^2e}\ \ \ \ !\ \log(2)\ \ \ _!\ ;\ \ _0=0;\qquad (3.32)$$

and

$$W\ \ \ \int_{-\infty}^{+\infty}\frac{d!dk_?^{D\ \ 2}}{!\ \ (2\ )^{D\ \ 2}}\frac{e^{i!t}\ e^{i\bar{k}_?\ \ \bar{z}}}{e^{!}\ \ \ 1}\cos\ 2!\ \log\ \sqrt{m^2+k_?^2e}\ \ \ \ 2!\ \log(2)\ \ \ 2\ _!\ :$$

Using (A.1):

$$W\ \ \ \int_{-\infty}^{+\infty}\frac{dk_?^{D\ \ 2}}{(2\ )^{D\ \ 2}}\frac{e^{i!t}\ e^{i\bar{k}_?\ \ \bar{z}}}{}2\log\ \sqrt{m^2+k_?^2e}\ \ =$$

$$=\ \frac{2}{}\int_0^{+jk_?j_{max}}\frac{jk_?j^{\frac{D\ \ 2}{2}}\ djk_?j}{(2\ )^{\frac{D\ \ 2}{2}}j\ \ \ \bar{z}j^{\frac{D\ \ 4}{2}}}J_{\frac{D\ \ 4}{2}}\ \ jk_?jj\ \ \ \bar{z}j\ \ \log\ \sqrt{m^2+jk_?j^2e}$$

$$\frac{2}{}\int_0^{+jk_?j_{max}}\frac{jk_?j^{\frac{D\ \ 2}{2}}\ djk_?j}{(2\ )^{\frac{D\ \ 2}{2}}j\ \ \ \bar{z}j^{\frac{D\ \ 4}{2}}}J_{\frac{D\ \ 4}{2}}\ \ jk_?jj\ \ \ \bar{z}j\ \ \log\ \ jk_?je\ \ \ :$$

Note that we consider only $jk_?j<jk_?j_{max}$, because in this case the approximation (3.32) works. Let us denote $e=j\ \ \bar{z}j\ \ \ \ \ \ 1$ and $jk_?j\ \ j\ \ K_?j=j\ \ \bar{z}j$:

$$W\ \ \ \ \frac{2}{}\frac{1}{(2\ )^{\frac{D\ \ 2}{2}}j\ \ \ \bar{z}j^{D\ \ 2}}\int_0^{+jk_?j_{max}}\ \ jK_?j^{\frac{D\ \ 2}{2}}\ djK_?jJ_{\frac{D\ \ 4}{2}}\ \ jK_?j\ \ \log\ \ jK_?j$$

$$\frac{2}{}\frac{1}{(2\ )^{\frac{D\ \ 2}{2}}j\ \ \ \bar{z}j^{D\ \ 2}}\int_0^{+jk_?j_{max}}\ \ jK_?j^{\frac{D\ \ 2}{2}}\ djK_?jJ_{\frac{D\ \ 4}{2}}\ \ jK_?j\ \ K_0\ \ jK_?j$$

$$\frac{2}{}\frac{1}{(2\ )^{\frac{D\ \ 2}{2}}j\ \ \ \bar{z}j^{D\ \ 2}}\ \ \frac{D\ \ \ 2}{2}\ \ 2^{\frac{D\ \ 4}{2}}:\quad (3.33)$$

This result coincide with (3.31), if $= \frac{2}{N}$.

3.3 Stress energy tensor in 2D

Here we complete the discussion of the massive scalar eld in 2D Rindler spacetime by examining the renormalized stress-energy tensor at various temperatures. As in the de Sitter space case we use Point-splitting method, see C. To set up the notations let us summarise the standard expression resulting from point splitting regularization in the Poincare invariant case $= 2$:

$$\langle T_{VV}\rangle_2 = \frac{t_V t_V}{4 \quad ^2}; \quad \langle T_{UU}\rangle_2 = \frac{t_U t_U}{4 \quad ^2};$$
$$\langle T_{VU}\rangle_2 = \langle T_{UV}\rangle_2 = \frac{e^{V \quad U}}{8} m^2 \quad _e + \log(m) + \log \sqrt{\quad t \ t} \quad ; \qquad (3.34)$$

where t is the vector separating the two points of the Wightman function (3.14).

The above expressions lead to the covariantly conserved stress energy tensor [52]:

$$\langle T \quad \rangle_2 = \frac{1}{4} m^2 [\quad + \log(m)] \, g \quad ; \qquad (3.35)$$

where is the Euler-Mascheroni constant. This is obviously related to the expectation value in Minkowski space by the coordinate transformations (3.7).

Similarly, for $= 2 = N$ point splitting regularization in (3.17) gives

$$\langle T \quad \rangle_{\frac{2}{N}} = \sum_{n=1}^{N \ 1} \frac{m^2}{4} e^{V \quad U} K_2 \quad 2me^{\frac{V \ U}{2}} \sin \frac{n}{N} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} +$$
$$+ \quad \frac{1}{4} m^2 [\quad + \log(m)] + \frac{m^2}{2} \sum_{n=1}^{N \ 1} K_0 \quad 2me^{\frac{V \ U}{2}} \sin \frac{n}{N} \quad g \quad ; \qquad (3.36)$$

where $K_0(x)$ and $K_2(x)$ are MacDonald functions. Violation of Poincare invariance is manifest.

Near the horizon this expression simpli es to:

$$\langle T \quad \rangle_{\frac{2}{N}} = \frac{1}{24} \quad N^2 \quad 1 \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + O(e^{V \quad U}); \qquad (3.37)$$

while at the spatial in nity it gives:

$$\langle T \quad \rangle_{\frac{2}{N}} \quad \frac{1}{4} m^2 [\ _e + \log(m)] \, g \quad ;$$

which coincides with the $= 2$ case. These two types of asymptotic behaviour of the stress energy tensor are regular. Furthermore, the second one does not depend on . On the other hand, the expectation value of the mixed components of stress energy tensor T diverge at the horizon. For generic values of , when both points in (3.27) are taken to the horizon we get (see appendix B)

$$W \ (X^+; X \quad) \quad \int_{1}^{1} \frac{d!}{!} \frac{e^{\frac{i!}{2}(V^+ + U^+ \quad V \quad U \ )}}{1 \quad e \ ^{!}} \sin \ ! \ \log(me^{\frac{(V^+ \ U^+)}{2}} = 2) + \arg \ (1 \quad i! \ )$$
$$\sin \ ! \ \log(me^{\frac{(V \ U \ )}{2}} = 2) + \arg \ (1 \quad i! \ ) \quad : \qquad (3.38)$$

The expectation value may be obtained by taking into account

$$\partial_{V_+}\partial_{V_-} W(X^+;X^-) = \int_{-\infty}^{\infty} \frac{d\omega}{4\pi}\,\frac{\omega}{1-e^{-\beta\omega}}\, e^{-i\omega(V^+-V^-)} =$$

$$= -\frac{1}{4\pi(V^+-V^-)^2} + \frac{\pi}{12\beta^2} = -\frac{t_V t_V}{4\pi^2} + \frac{1}{24\pi} + \frac{\pi}{12\beta^2}: \qquad (3.39)$$

At the horizon for arbitrary temperatures we get

$$\langle T_{\mu\nu}:\rangle = \frac{1}{24\pi}\left(\left(\frac{2\pi}{\beta}\right)^2 - 1\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + O(e^{V-U}) \qquad (3.40)$$

to be compared with (3.37).

4 Conclusions and Acknowledgement

One of the main obtained results is that for non-canonical temperature ($\beta \neq \beta_C$) propagators do not have standard Hadamard behaviour near the horizon. Furthermore, only the propagator with the canonical temperature respects the de Sitter isometry or Poincaré invariance, correspondingly. I.e. only in such a case propagators are functions of geodesic distances. Moreover there is non-zero regularized stress energy tensor in the horizon limit, when $\beta \neq \beta_C$. This result, generally speaking, may lead to the drastic deformation of the metric near the horizon, as in [53]. The natural question arises: how to define a gas of exact modes with arbitrary temperature in de Sitter space?

Also, it should be noted that Cauchy surfaces in the Rindler or the static de Sitter spaces are not equivalent to those in the geodesically complete global spaces (Minkowski space and the global de Sitter space). Hence initial data on such "incomplete" surfaces cannot determine uniquely dynamics of fields in the complete global spaces. In particular, this is related to the fact that the maximally symmetric states are mixed. And different choices of initial Cauchy surfaces may lead to substantially different dynamics [3, 5].

It is also interesting that the long-time behavior of the propagator in the de Sitter space changes in a leap at the point $\beta = 4\pi$. Moreover, if we pay attention to the dependence of the correlation length on the temperature, the result turns out to be opposite to our intuition: infinite temperature limit $\beta = 0$ corresponds to the finite correlation length. Taking into account the result of [54], where is shown that the Debye screening is absent for the canonical temperature, one can pose the question of the physical meaning of temperature in these cases. Namely we see several examples when intuition from thermodynamics in the flat space leads to expectations that do not coincide with calculations in curved spaces. This in turn means that, for example, the process of thermolization in the de Sitter space may turn out to be more complicated than we expect [55]. An example is the previously noted infrared features of loop corrections: [1–5].

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A Leading infrared contribution

The behavior of various Wightman functions discussed above at the horizons is governed by the integral of the form:

$$\int_{-\infty}^{+\infty} \frac{d\omega}{\omega + i} \frac{e^{i\omega}}{e^{(\omega + i)} - 1}; \quad \text{where } |\ | \ll 1.$$

The choice of the shifts of the poles here reproduces the results in the case $2 N$ but it can also justified by general distributional methods. The contour in closed in the upper half-plane for positive values of and in the lower half for negative ones. In the first case the double pole at $\omega = -i$ does not contribute. Contributions from other poles are suppressed. For negative the leading contributions in the limit $\omega \to \infty$ comes from the double pole at $\omega = -i$:

$$\int_{-\infty}^{+\infty} \frac{d\omega}{\omega + i} \frac{e^{i\omega}}{e^{(\omega + i)} - 1} \approx \begin{cases} 0 & \text{if } > 0 \\ \frac{2}{} & \text{if } < 0 \end{cases} \quad \text{as } |\ | \ll 1. \tag{A.1}$$

the answer depends on the sign of .

B Completeness relation of Associated Legendre Functions on the cut

The orthogonality relation of Legendre functions can be found in [32]. Here we provide an explicit (formal) calculation "completeness" relation of Legendre functions to calculate the Canonical Commutation Relations (2.10) which, by introducing $\cos = \tanh x = u$, we rewrite as follows:

$$\sin_1 \sin_2 (\cos_1 - \cos_2) = \int_{-\infty}^{\infty} \frac{\omega\, d\omega}{4 \sinh(\ )} \left(\frac{1}{2} + i - i\omega\right) \left(\frac{1}{2} - i + i\omega\right) \Big[P^{i\omega}_{-\frac{1}{2}+i}(\cos_1) P^{i\omega}_{-\frac{1}{2}+i}(\cos_2) + P^{i\omega}_{-\frac{1}{2}+i}(-\cos_1) P^{i\omega}_{-\frac{1}{2}+i}(-\cos_2) \Big]. \tag{B.1}$$

Using the holomorphic plane waves introduced in Sec. (2.4) we get the following integral representation for $P^{i\omega}_{-\frac{1}{2}+i}(\cos\ )$:

$$P^{i\omega}_{-\frac{1}{2}+i}(\cos\ ) = \frac{i\ (\frac{1}{2} + i\ )}{2\ (\frac{1}{2} + i - i\omega)} \int_{-\infty}^{\infty} dt e^{i\omega t} f(t;\ ) \tag{B.2}$$

where we set

$$f(t;\ ) = (\ (0)\ Z(t - i;\ ))^{-\frac{1}{2} - i} = [\cos + \sin \sinh(t - i\ )]^{-\frac{1}{2} - i}; \tag{B.3}$$

$$f(t;\ ) = (f_+(t;\ ) - f(t;\ )). \tag{B.4}$$

$P^{i\omega}_{-\frac{1}{2}+i}(\cos\ )$ is therefore the Fourier transform of the discontinuity of the holomorphic plane waves on the real de Sitter manifold. Let us insert (B.2) in Eq. (B.1); let us consider for instance the first term on the rhs of Eq. (B.1). By performing the integration over ω we get

$$(B.1) = \frac{i}{16 \sinh^2} \int_{-\infty}^{\infty} dt\, [(\partial\ f(t;_1))\ f(t;_1) - f(t;_1) \partial\ f(t;_2)] + $$

$$\frac{i}{16\ \sinh^2}\int_{1}^{1} dt\,[(\partial\ f\,(t;\quad{}_1))\ \ f\,(t;\quad{}_1)\quad f\,(t;\quad{}_1)\partial\ \ f\,(t;\quad{}_2)\ ] =$$

$$= \frac{i}{16\ \sinh^2}\sum_{k=}\int_{1}^{1} dt\,[(\partial f_k(t;\ {}_1))\, f_k(t;\ {}_1)\quad f_k(t;\ {}_1)\partial f_k(t;\ {}_2)\ ] +$$
$$\frac{i}{16\ \sinh^2}\sum_{k=}\int_{1}^{1} dt\,[(\partial f_k(t;\quad{}_1))\, f_k(t;\quad{}_1)\quad f_k(t;\quad{}_1)\partial f_k(t;\ {}_2)\ ]: \qquad \text{(B.5)}$$

In the second step we used the analyticity properties of the plane waves; this simpli cation is valid in the two-dimensional spacetime and in any even dimensional spacetime as well. By introducing the Mellin representation of the plane wave:

$$f\ \ (t;\ \ ) = \frac{e^{\frac{i}{2}(\frac{1}{2}+i\ \ )}}{(\ \frac{1}{2}+i\ \ )}\int_0^1 du\ u^{\frac{1}{2}+i}\ e^{\,iu(\cos\ \ +\sin\quad \sinh(t\ \ i\ \ ))};\qquad 0<\ \ <\ \ ; \qquad \text{(B.6)}$$

a few easy integrations show the validity of Eq. (B.1) and the completeness of the modes.

C Point-splitting method of covariant regularization of stress energy tensor

Let us introduce the light-cone coordinates of the static patch:

$$V = t + x;\qquad U = t\quad x;$$
$$ds^2 = \frac{1}{\cosh^2(\frac{V\ \ U}{2})}dU\,dV \quad C(U;V)dU\,dV: \qquad \text{(C.1)}$$

Firstly regularization method should be discussed. We perform the actions in the same way as in [25]. Let us consider general metric in two dimensions:

$$ds^2 = C(u;v)dudv$$

and the following stress-energy tensor:

$$T\qquad \partial'\partial\ '\qquad \frac{1}{2}g\ \ g\quad \partial'\partial\ ':$$

The key idea of well-know point-splitting regularization method [48] is as follows: we divide point x into two points x and the average of the derivatives is replaced by the derivatives of the Wightman function:

$$T\ \ (x)\ !\ \ D\ \ h\hat{'}(x^+)\hat{'}(x\ \ )i\,;$$

then connect them along a curve, which is the geodesic line between them and the starting point x:

$$(x^+)\ \ (\ \ ) = x\ \ +\ \ t\ \ +\frac{1}{2}\ {}^2a\ \ +\frac{1}{6}\ {}^3b\ \ + :::;\qquad (x\ \ )\ \ (\ \ ) = (\,x^+)\ \ (\quad);$$

where coe cients t ; a ; b have the following relations:

$$a\ \ =\qquad t\ \ t\ \ ;\quad b\ \ =\qquad (a\ \ t\ \ + t\ \ a\ \ )\quad t\ \ \partial\qquad t\ \ t\ \ :$$

Also we need the parallel transport matrixe ( ), de ned as:

$$\frac{de}{d} + \qquad \frac{dx}{d}e\ \ = 0;\quad e\ (\quad = 0) = \qquad :$$

Finally:

$$h\hat{T} \quad i = h@' (x^+)@' (x \quad)i \quad e^+ \quad e \quad \frac{1}{2} g \quad g \quad e^+ \quad e \quad ;$$

where $e \quad = e \,(\quad = \quad)$ and:

$$e \quad = \quad + \quad t \quad + \frac{1}{2} \, {}^2 a \quad + :::$$

where

$$t \quad = \quad t \; ; \quad a \quad = \quad t \; t \; + \quad t \; t \quad t \; t \; @ \quad :$$

And:

$$h T \quad i = \frac{1}{4 \quad {}^2 (t \; t \;)} + \frac{R}{24} \quad \frac{t \; t}{t \; t} \quad \frac{1}{2} g \quad + \quad ;$$

nally terms which do not depend on and gives the following answer:

$$h : T \quad :i = \quad + \frac{R}{48} g \quad ; \tag{C.2}$$

with

$$\begin{aligned} {}_{uu} &= \quad \tfrac{1}{12} C^{1=2} @_u^2 C^{1=2} + \text{state dependent terms;} \\ {}_{vv} &= \quad \tfrac{1}{12} C^{1=2} @_v^2 C^{1=2} + \text{state dependent terms;} \\ {}_{uv} &= \quad {}_{vu} = 0 : \end{aligned} \tag{C.3}$$

Also we need to clarify what is "state dependent terms". During regularization we paid attention only to ultraviolet divergences. In fact the stress-energy tensor contains nite terms that depend on the state. For example for two dimensional gas of massless particles with number of particles n_{jkj} and, say T_{uu} component of stress-energy tensor, we have:

$$@_{u^+} @_{u} \; W = \int_0^{+1} \frac{d!!}{4} \left[e^{i! (u^+ \quad u \;)} n_{jkj} + e^{\quad i! (u^+ \quad u \;)} (n_{jkj} + 1) \right]$$

$$\frac{1}{4} \frac{1}{(u^+ \quad u \;)^2} + \int_0^{+1} \frac{d!!}{2} n_{jkj} \; :$$

The rst term should be regularized according to the scheme described above, while the second is nite and named as "state dependent terms" in (C.3).

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